**2.5 Inverse Matrices**

1. Definition: and

Not all matrices have inverses.

2. Notes:

(1) Note 3: If A is invertible, the one and only one solution to is

(2) Note 5: A matrix is invertible if its determinant is not zero.

e.g : Determinate for 2 by 2 matrix is . For example, is not invertible.

3. Inverse of a product.

(1) The inverse of product is , in **reverse order**.

Example: Inverse of an elimination matrix:

Subtracts 5 times row 1 from row 2, E: , adds 5 times …,

4. Calculate by Gauss-Jordan Elimination.

Definition: Multiply by to get

Example 4: (echelon 排成梯队, called augmented matrix)

,

**2.6 Elimination = Factorization: A=LU**

Example:

Forward from A to U:

Back from U to A:

If A is a 3 by 3 matrix, multiply by , Elimination ends with the upper triangular U.

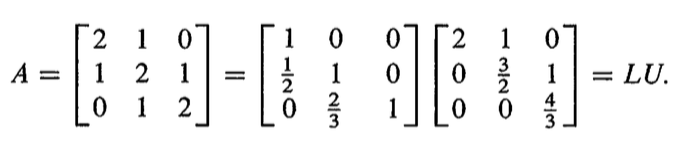
Becomes Which is A=LU

L: **lower triangular production of inverses.**

As with 3 by 3 matrices, row 3 of A=LU:

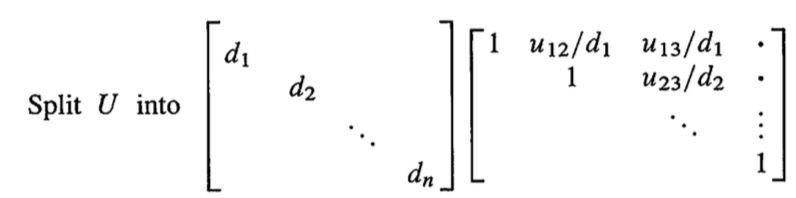
**(row 3 of A) = (row1 of U) + (row2 of U)+ 1(row3 of U)**

Example:gg



Better balance:

Divide U by diagonal matrix D that contains the pivots , leaves matrix with 1’s on the diagonal:



Then the triangular factorization can be written A=LU or A=LDU

To solve Ax=b , we solve Lc=b (forward), after c is resolved, then solve Ux=c (backward)

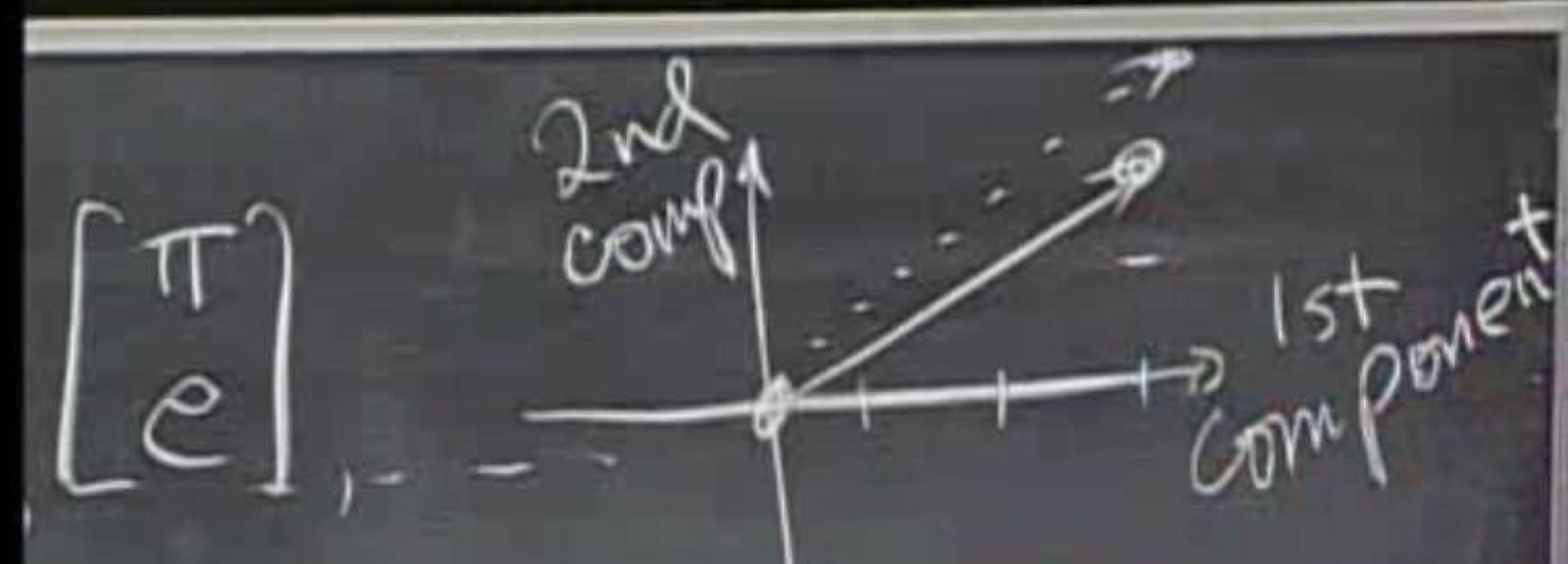
The cost of elimination:

Factor: multiplications and subtractions on the left side.

Solve: multiplications and subtractions on the right side.

**Lecture 5: Transpose, Permutations, spaces**

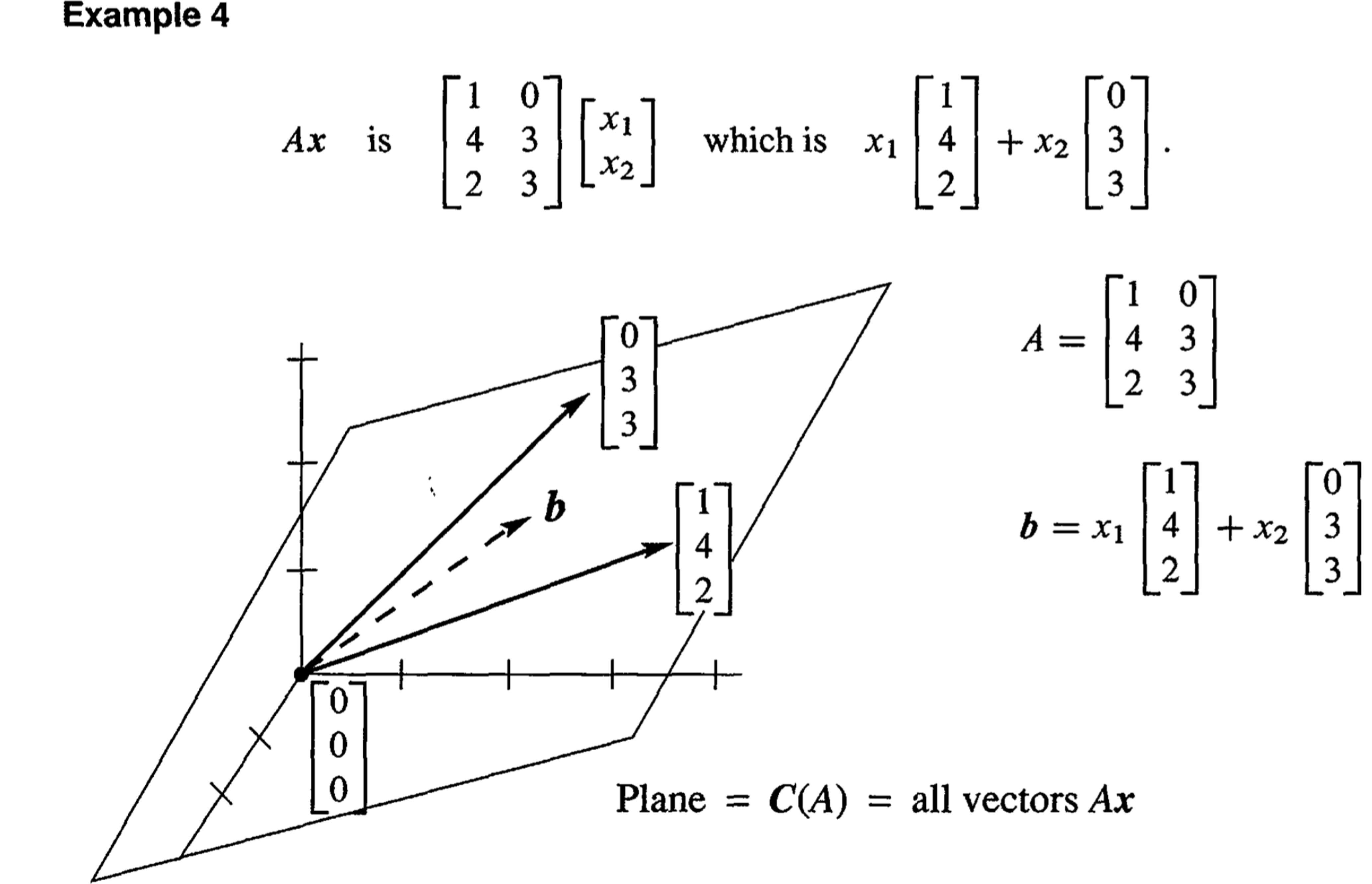
1. Permutation: P, row exchange
   1. PA=LU, P gets the row into the good order.
   2. Permutation = identity matrix with reordered rows.
   3. , counts n by n permutations.
2. Transpose , Symmetric : , is **always symmetric**.
3. Vector spaces:
   1. Example: = all two dimensional real matrices. Spaces: add, multiply and get combinations , x-y plan as below:



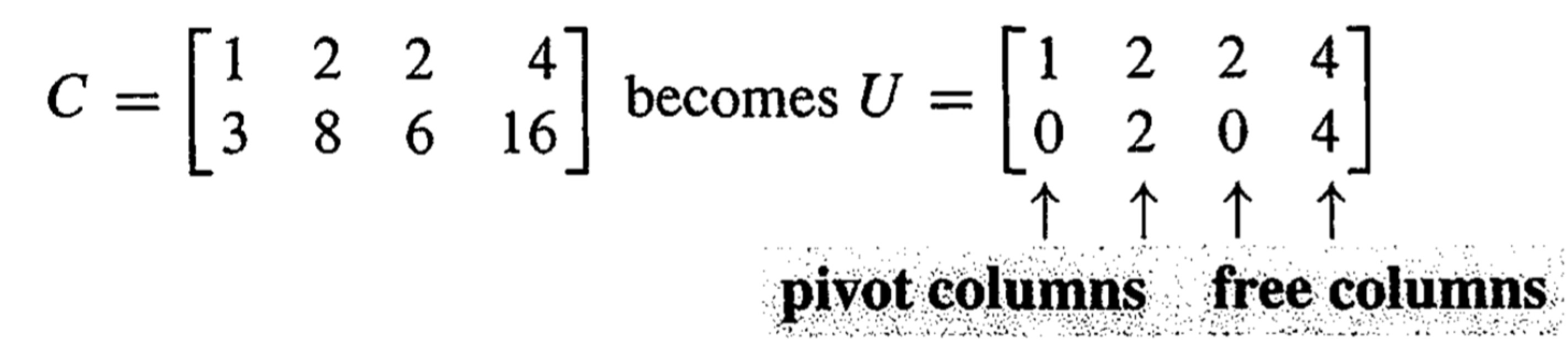
* 1. Space consists of all column vectors v with **n components**.
  2. Subspaces
     1. Example: , choose a plain t**hrough the origin (0,0,0),** the plain is a vector space, and it is a subspace of , although it looks like a 2 dimensional space.
     2. A subspace of a vector space is a set of vectors (**including 0**) satisfy:
        1. is in the subspace,
        2. is in subspace.
        3. , all linear combinations stay in subspace .

**Lecture 6: Column space and nullspace**

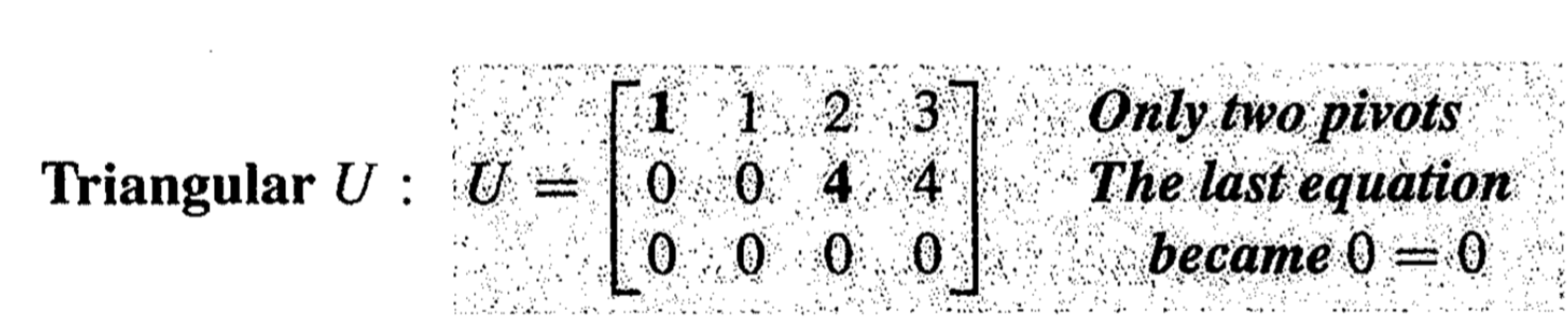
1. Goal of column space , in order to solve Ax=b, **those b’s** form the “column space” of A.
2. Column space: Column space consists of all linear combinations of columns. Combinations are **possible vectors Ax**. They fill the column space C(A). IMPORTANT: why the combination is Ax other than x? Because Ax is actually a combination with a factor x. **x is a variable here**.
3. The system Ax=b is solvable if and only if b is in column space of A.
4. Example:



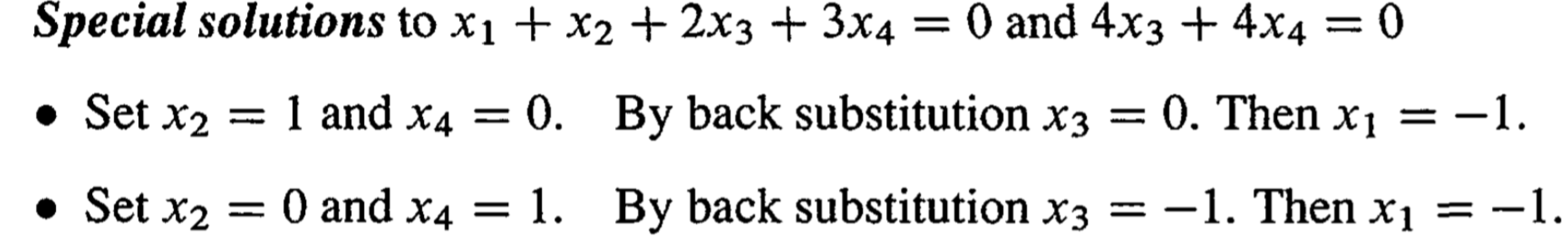
1. Nullspace of A: solving Ax=0
   1. The nullspace consists of all combinations of **special** **solutions** of Ax=0. Note here solutions is not same to them in column vector : combinations of Ax.
   2. The solution vector x has n components in , so the nullspace is a subspace of . The column space is a subspace of .
   3. Pivot columns and free columns: Elimination will produce



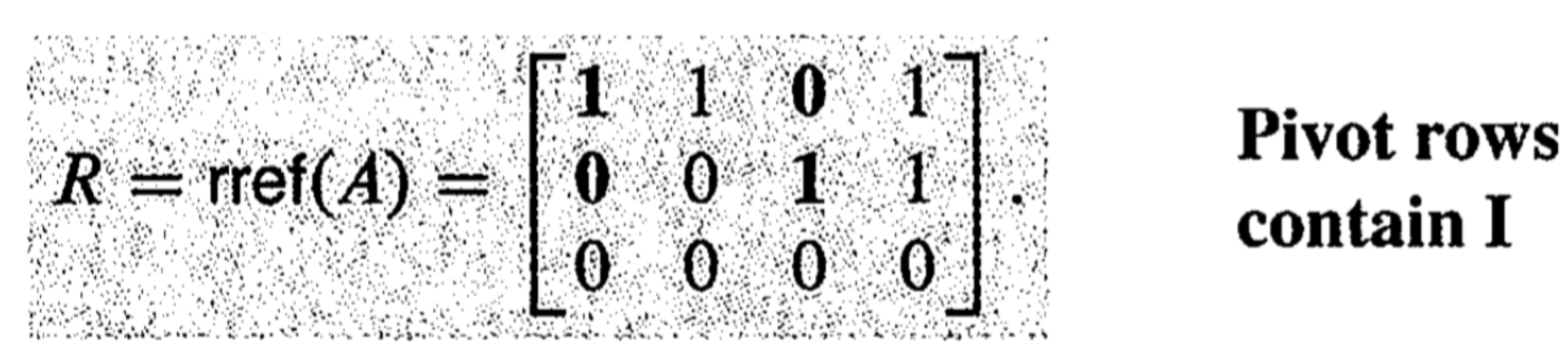
* 1. In order to solve Ax=0, take two steps below:
     1. Forward elimination takes A to a triangular U or reduced form R
     2. Back substitution in Ux=0 or Rx=0 produce x.
     3. For example :



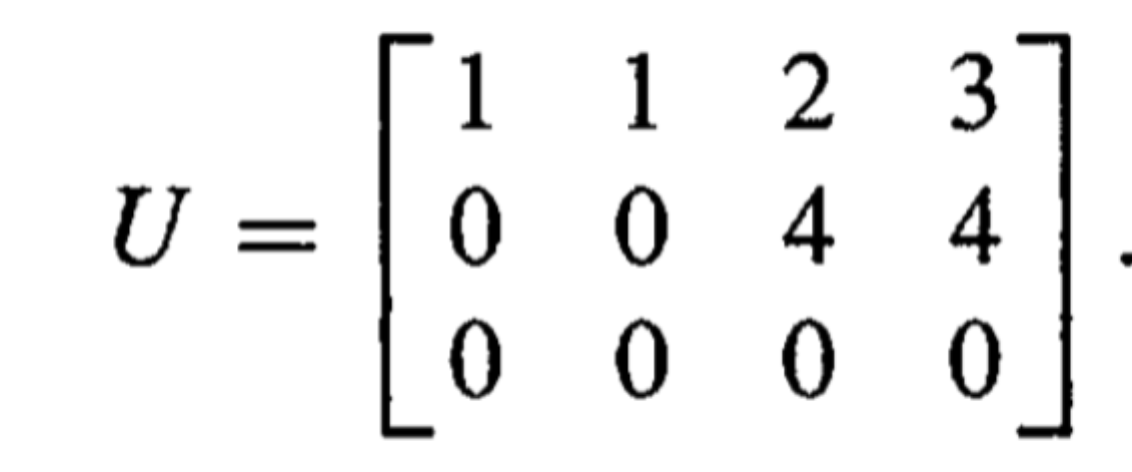
pivot variable x1 and x3, free variable is x2 and x4 . Free variable **can be given any values**.



* 1. Reduced Row Echelon Matrix R
     1. Pivot rows contains 1.



from upper triangular



* 1. If n>m, A has at least one column without pivots, giving a special solution , so there are nonzero vectors x in the null space of this rectangular matrix A.